Improving the $k - \varepsilon$ turbulence model for simulation of atmospheric boundary layer flow

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Long abstract

Simulation of atmospheric boundary layer (ABL) flow is a topic of increasing interest within the computational fluid dynamics community. An accurate description of the mean turbulent flow within the first few hundred metres of the atmosphere is especially pertinent in the analysis of pollutant dispersion and in determining site suitability for wind energy applications. Although large-eddy simulation is becoming increasingly popular, the RANS approach remains the practical tool of choice for such work. Within this context, the most popular closure scheme is by far the $k - \varepsilon$ turbulence model.

The generally accepted practice for numerically reproducing freestream flow conditions using a RANS/$k - \varepsilon$ approach is to apply the boundary conditions proposed by Richards and Hoxey (1993) in which wall functions are explicitly based on an aerodynamic roughness length instead of an equivalent sand grain roughness. The weaknesses of the latter method have been recently documented by Blocken *et al* (2007). The success and popularity of the former is due to its ability to yield unvarying distributions of velocity and turbulent properties for horizontally homogeneous conditions. Its only apparent weakness concerns the turbulent kinetic energy, k, in the region immediately adjacent to solid boundaries where it is consistently overestimated (see figure 1).

Although for flow simulations over flat terrain this does not necessarily pose a problem, as k quickly decreases to its freestream value away from the ground, there are at least two situations where such behaviour may introduce significant

errors. First, for flow in complex terrain, this local error may be propagated to higher elevations by the variable orography. Second, changing surface conditions cause the formation of internal boundary layers which may be sensitive to the near-wall k distribution.

The spike in k is often attributed to an overestimation of the turbulence production term in the first few cells nearest the wall. More precisely, it is largely due to an imbalance between production and dissipation terms stemming from the fact that both are dependent on quantities that vary rapidly as $z \to 0$. To remedy this problem, wall damping functions are proposed, in the spirit of low-*Re* $k-\varepsilon$ models, to adjust the integrated source terms in the k and ε transport equations to account for discretization errors introduced by the numerical method. Furthermore, it is proposed to correct Laplacian terms in the ε and momentum equations by replacing the usual linear discretization schemes with other schemes inspired by analytically derived near-wall distributions of ε and U.

The implementation of the source term corrections in OpenFOAM simply requires modifying the existing $k-\varepsilon$ turbulence equations as shown below. Corrections to Laplacian terms (implicit and explicit) require the definition of new interpolation and snGrad schemes. The wall functions introduced are formulated purely in terms of grid geometry and do not increase the computational burden.

$$\frac{\partial k}{\partial t} + \nabla \cdot k\vec{U} = \nabla \cdot \left(\frac{\nu_t}{\sigma_k} \nabla k \right) + \frac{G_k}{f_{\varepsilon 2}} - f_{\varepsilon 2}\varepsilon \tag{1}$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \varepsilon\vec{U} = \nabla \cdot \left(f_{\varepsilon 1} \frac{\nu_t}{\sigma_\varepsilon} \nabla \varepsilon \right) + C_{\varepsilon 1} G_k \frac{\varepsilon}{k} - f_{\varepsilon 2}^2 C_{\varepsilon 2} \frac{\varepsilon^2}{k} \tag{2}$$

For the sake of simplicity, the present analysis is limited to two-dimensional flow and a structured grid. The test case and grid recently used by Hargreaves and Wright (2007) to perform a similar analysis is also used here to demonstrate the effect of the proposed corrections. As shown in figure 2, simulations in an empty domain yield outlet profiles which match the analytical solutions to a high degree.

Although the aim of this work is to identify the source of and correct numerical errors near solid boundaries that arise when using the standard $k-\varepsilon$ turbulence model for ABL flow, the findings may also be useful for other RANS closure schemes that involve variables that vary rapidly near walls (for example, RNG k–ε, k–ω, etc.). Future work will be aimed at adapting the proposed corrections for flow over complex terrain and arbitrarily unstructured grids.

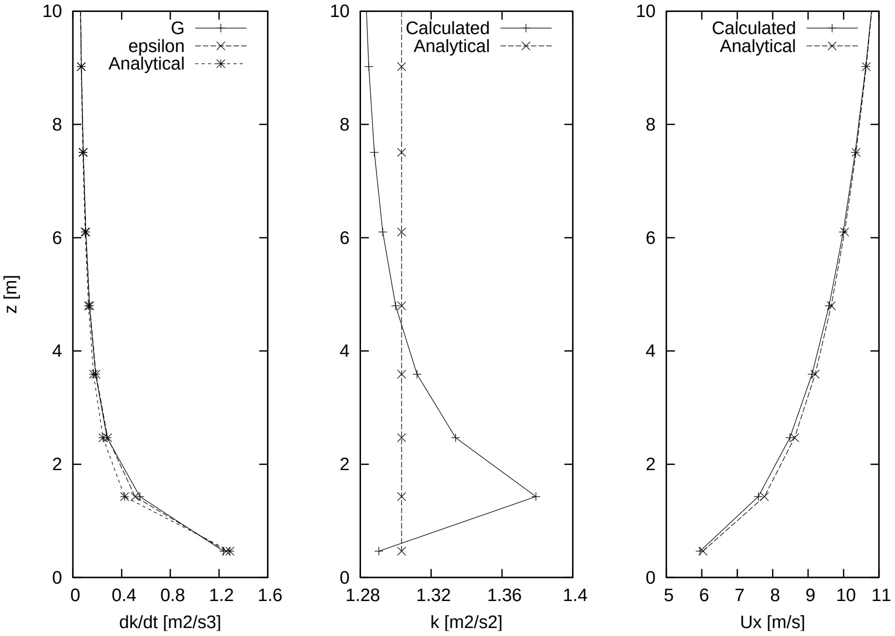

Figure 1: Typical RANS/$k-\varepsilon$ flow solution for homogeneous surface conditions

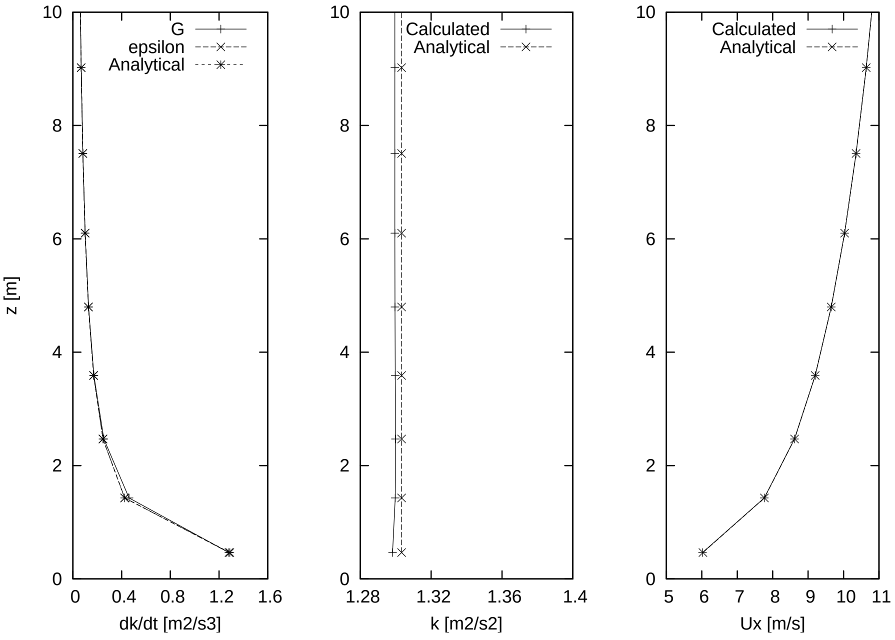

Figure 2: Flow solution with proposed corrections